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MOMENTUM AND HEAT TRANSFER IN TWO-PHASE BUBBLE FLOW—I

THEORY

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Abstract—A theory is proposed which describes the transfer process of momentum and heat in a two-phase bubble flow in channels. The eddy diffusivity to express the turbulent structure of the liquid phase is subdivided into the two components, one for the inherent wall turbulence independent of bubble agitation and the other for the additional turbulence caused by bubbles. On the basis of the theory, the velocity profile and the frictional pressure gradient for a given flow can be predicted when its void fraction profile is known. Furthermore, when a uniform heat flux is added to the system, its temperature distribution and heat transfer coefficient can be determined. A method for the numerical calculation of these parameters is also presented.

1. INTRODUCTION

Two-phase bubble flow is commonly defined as a flow pattern in which the gas phase is distributed within a liquid continuum in discrete bubbles smaller than the characteristic dimension of the channel (e.g. pipe radius), and is encountered in a wide range of industrial plants such as steam generators, bubble columns, heat exchangers for liquified natural gas and various gas–liquid pipeline systems. Understanding of the flow mechanism of this regime is not yet entirely satisfactory though several models have been presented concerning the cross-sectional velocity distribution: Bankoff (1960), Levy (1963), Brown & Kranich (1968) and Beattie (1972).

To predict momentum and heat transfer process of bubble flow it is of importance to elucidate turbulent structure of the continuous liquid phase, which may result in how to describe the contribution of bubble existence to the flow characteristics. As a first step toward the flow structure modeling, the authors have proposed an analysis of the momentum transfer in the previous study (Sato & Sekoguchi 1975), in which the turbulent shear stress in bubble flow is subdivided further into the two components, one due to the inherent liquid turbulence independent of relative motion of bubbles and the other due to the additional turbulence caused by bubble agitation. On the basis of this idea an attempt was made to predict theoretically the liquid velocity distribution in a vertical pipe when its void fraction profile was given. Then, in the core region of the flow, the validity of the analysis was confirmed by comparing the calculated velocity distributions with the experimental data.

However, since little consideration was given to the region close to the wall in the previous study, it was unable to make practical use of the analysis for the prediction of frictional pressure gradient. The first objective of this report is then to improve the analysis with special emphasis on the description of the wall region so as to be capable of predicting not only the liquid velocity distribution over the entire cross section but also the frictional pressure gradient.

Analytical models for heat transfer mechanism of bubble flow are practically nonexistent. It may be expected that, regarding bubble flows, the above-mentioned treatment for momentum transfer is also applicable to heat transfer problem, i.e. the additional turbulent heat flux is assumed to be subdivided into the two components, one due to the inherent liquid turbulence and the other caused by bubble agitation. The second objective is then to present an analysis which is able to describe the heat transfer process by means of such treatment. The result of the analysis leads to theoretical calculation of the liquid temperature distribution and heat transfer coefficient of diabatic bubble flows.

2. MOMENTUM TRANSFER

2.1 *Basic equations*

The present study is concerned with two-dimensional bubble flow, a flow in a vertical pipe or between two parallel flat walls. The coordinate system is shown in figure 1. The x-axis is parallel to the main flow direction, and y- and r-axes are normal to it, measured from the wall and the channel center respectively. At any position (x, y) of the channel, the liquid flow has velocity components (u_L, v_L) and a density ρ_L. Fluctuation of the liquid velocity from the mean may be split further into the two parts on the assumption that there are two kind of turbulence in the liquid phase independent of and dependent on bubble agitation, designated as (u', v') and (u'', v'') respectively. According to the previous study (Sato & Sekoguchi 1975), derivation of the basic equations pertinent to flow prediction is outlined in this paragraph.

If it is assumed that gas bubbles can be treated as mere voidages, no momentum transfer takes place in the gas phase and thus a knowledge of flow properties of liquid phase is sufficient to describe the flow. When the momentum equation in the x-direction is written in terms of mean liquid velocity and fluctuations and furthermore averaged with respect to time, rearrangement of the resulting equation allows the shear stress τ for liquid phase to be identified as

$$\tau = \mu_L \frac{d\bar{u}_L}{dy} - \rho_L \overline{u'v'} - \rho_L \overline{u''v''} \qquad [1]$$

where μ_L is the viscosity of the liquid and the overbar designates time-averaging. The first term on the r.h.s. of [1] represents the effect of viscosity on the mean flow whereas the second and the third terms are the additional stresses arising from the two components of liquid turbulence independent of and dependent on bubble agitation. Taking the existence of void into consideration, the total shear stress at any point can be written as follows:

$$\tau = (1-\alpha)\left(\mu_L \frac{d\bar{u}_L}{dy} - \rho_L \overline{u'v'} - \rho_L \overline{u''v''}\right) \qquad [2]$$

where the factor $(1-\alpha)$ means the probability of the existence of liquid phase, which can be regarded as the time-averaged volume fraction occupied by the liquid at the point.

It may be convenient to introduce the eddy diffusivities ϵ' and ϵ'' for the two additional stresses by putting

$$-\rho_L \overline{u'v'} = \rho_L \epsilon' \frac{d\bar{u}_L}{dy} \qquad [3a]$$

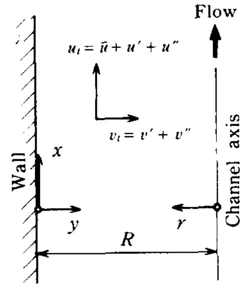


Figure 1. Flow system.

$$-\rho_L \overline{u''v''} = \rho_L \epsilon'' \frac{d\bar{u}_L}{dy}. \quad [3b]$$

Substituting [3a] and [3b] into [2] yields

$$\tau = \rho_L(1-\alpha)(\nu_L + \epsilon' + \epsilon'')\frac{du_L}{dy} \quad [4]$$

where ν_L is the kinematic viscosity of the liquid, and the overbar above the velocity has been omitted. For rewriting [4] in the dimensionless form, the following variables are defined;

$$u_L^+ = u_L/u_L^* \quad [5]$$

$$y^* = y/R \quad [6]$$

$$\tau^* = \tau/\tau_w \quad [7]$$

where u_L^* is the friction velocity defined as $u^* = \sqrt{(\tau_w/\rho_L)}$, τ_W the wall shear stress, and R the radius of a pipe or the half width of a channel with parallel flat walls. Then, [4] becomes

$$\frac{du_L^+}{dy^*} = \frac{\tau^*}{(1-\alpha)(\nu_L + \epsilon' + \epsilon'')/Ru_L^*}. \quad [8]$$

In the previous study the Reichardt's formula (Reichardt 1951) was used for the one eddy diffusivity ϵ',

$$\epsilon' = \frac{\kappa R u_L^*}{6}(1 - r^{*2})(1 + 2r^{*2}) \quad [9]$$

in which κ is the mixing length constant, and r^* the dimensionless radial distance, $r^* = r/R$. Equation [9] is known to be valid for the region of $r^* < 0.9$. As for the other eddy diffusivity ϵ'', the following formula has been proposed taking account of the "drift" phenomena of liquid particles due to relative motion of gas bubbles:

$$\epsilon'' = k_1 \alpha \left(\frac{d_B}{2}\right) U_B, \quad [10]$$

where k_1 is an empirical constant and d_B and U_B are the mean diameter and relative velocity of the bubbles respectively. In the core region at a distance from the wall, [9] and [10] have been confirmed to be reasonable by comparing the predicted velocity profile with the experimental data.

Equation [10] is of similar form to the well-known virtual kinematic viscosity of a free turbulent flow such as a wake behind a solid body. According to the Prandtl's hypothesis (Prandtl 1942), such the virtual kinematic viscosity is expressed as

$$\epsilon = \kappa_1 b u_{1\max} \quad [11]$$

where b is the width of the mixing zone and $u_{1\max}$ the maximum deficit velocity. The reason why [10] involves a parameter α (local void fraction) in contrast to [11] is that, in a bubble flow, the additional turbulence at a given point may arise from the transit of bubbles there; in other words, this parameter corresponds to the probability of the vortex generation. From the form of [2] and the above-mentioned similarity between [10] and [11], it should be emphasized again that the proposed theory is based on the assumption that the turbulence structure of the liquid phase consists of superposition of the two independent mechanisms, the wall turbulence and the free turbulence due to wakes of bubbles.

2.2 *Shear stress distribution*

If two coaxial fluid cylinders of any radius r and pipe radius R are considered in a vertical pipe, the ratio of the shear stress τ acting on the circumferential surface at r to the wall shear stress τ_w can be determined from the force balance,

$$\tau^* = \frac{\tau}{\tau_w} = \left(1 \mp B \int_0^1 \alpha r^* \, dr^*\right) r^* \pm \frac{B}{r^*} \int_0^{r^*} \alpha r^* \, dr^* \qquad [12]$$

in which B is the dimensionless parameter, $B = gR/u_L^{*2}$.

In a case of flow between two parallel flat walls, the shear stress ratio becomes

$$\tau^* = \left(1 \mp B \int_0^1 \alpha \, dr^*\right) r^* \pm \int_0^{r^*} \alpha \, dr^* . \qquad [13]$$

The upper and the lower signs in both [12] and [13] should be used for an upward flow and a downward flow, respectively. Furthermore, when a horizontal flow between parallel flat walls is concerned, the ratio is of course written as $\tau^* = r^*$.

2.3 *Value of constant* k_1

An experiment has been performed to determine the empirical constant k_1 of [10]. In the previous study k_1 was taken to be unity based on a few experimental velocity profiles in a pipe. However, there has been room for further discussion on this value because the applied data had been obtained from air–water bubbly flows alone in which the effects of both the wall turbulence and the bubble agitation were seen to be coexistent and then ϵ' and ϵ'' should be comparable in the basic equation [8]. In order to examine it in more detail and to recommend a reliable value, it is expected that bubble flows in the absence of the wall turbulence offer the available information about bubble agitation alone.

It may be reasonable to postulate that, even in a two-phase bubble flow, the wall turbulence is suppressed when the liquid Reynolds numbers (based on bulk velocity) is lower than the critical value similar to that for single-phase flow;

$$\mathrm{Re}_L = \frac{j_L D}{(1-\hat{\alpha})\nu_L} < 2300 \qquad [14]$$

where j_L is the volumetric flux density of the liquid phase, D the pipe diameter and $\hat{\alpha}$ the mean void fraction. Under such a situation, [8] becomes

$$\frac{du_L^+}{dy^*} = \frac{\tau^*}{(1-\alpha)(\nu_L + \epsilon'')/Ru_L^*}. \qquad [15]$$

In the present experiment 62 wt per cent aqueous glycerol solution was used as the liquid phase to attain the above condition [14] even at high bulk liquid velocities enough for the velocity profile and the frictional pressure drop to be measured more accurately. Such a solution has a kinematic viscosity about ten times of that of pure water at room temperature. Air was the gas phase. The test section was a vertical pipe of 26.0 mm i.d. and 5.6 m long. The liquid velocity and the void fraction profiles were measured at a downstream distance of 4.3 m from an air–liquid mixer by means of a hot-film anemometer and an electrical resistivity probe method, respectively. As for the bubble relative velocity U_B in [10], the terminal velocity of bubbles in the still liquid were determined from photographs taken with multiflash exposure using an electronic stroboscope. For the further details of the experimental facilities and method, the following report in this series (Sato *et al.* 1981) should be referred.

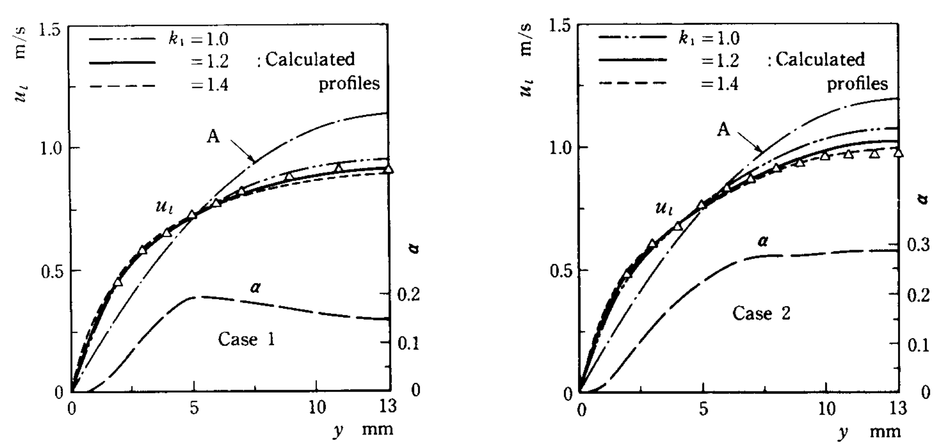
The two typical results for the measurement of liquid velocity and void fraction profiles are shown in figures 2(a) and 2(b); the data points pertain to the velocity u_L and the dashed curve does to the void fraction profile, respectively. The flow parameters of these runs are presented in table 1. In each figure the predicted velocity profiles are also drawn, calculated from [15] together with [10] and [12] taking the empirical constant of [10] to be $k_1 = 1.0$, 1.2 and 1.4 in order. (The method for the numerical calculation will be presented later in section 4.) Inspection of the figures indicates that there are few air bubbles near the wall in such viscous bubble flows, and the predicted velocity profile is in good agreement with the experimental data for a value of $k_1 = 1.2 \sim 1.4$.

The value of wall shear stress τ_w of these flows obtained from the numerical calculation is also placed in table 1. From a comparison of the calculated and measured wall shear stresses, it is found that the calculated values agree within 4 per cent with the measured value in a case of $k_1 = 1.2$. Accordingly, the empirical constant k_1 can be recommended as 1.2 when the terminal velocity in the still liquid is applied to the bubble relative velocity U_B.

The curve A in each figure is a velocity profile for laminar flow corresponding to the bulk velocity of $j_L/(1-\hat{\alpha})$, calculated by

$$u = \frac{2j_L}{(1-\hat{\alpha})}\left\{1-\left(\frac{r}{R}\right)^2\right\}. \qquad [16]$$

As can be seen from the figures, this laminar velocity profile deviates from the data points. The deviation can be attributed to the existence and agitation of air bubbles.



(a) $Re_l = 1470$ (b) $Re_l = 1546$

Figure 2. Comparison of the calculated liquid velocity distributions with the experimental data for viscous bubble flows, $Re_L = 1470$ and 1546. (Corresponding flow parameters are listed in table 1.) Curves u_L: calculated velocities by taking $k_1 = 1$, 1.2 and 1.4. Curve A: velocity profile for the laminar flow given by [16]. Curve α: measured void fraction profile.

Table 1. Flow parameters for the experiment shown in figure 2

Case	j_ℓ [m/s]	j_g [m/s]	$\hat{\alpha}$ [-]	Re_ℓ [-]	$\tau_{w,Exp}$ [Pa]	$\tau_{w,Cal}$ [Pa] $k_1=1$	1.2	1.4	Remarks
1	0.50	0.10	0.119	1470	3.00	2.81	3.10	3.47	Fig. 2(a)
2	0.50	0.15	0.159	1546	3.07	2.59	2.97	3.35	Fig. 2(b)

Fluids : Air and 62 wt % aqueous glycerol solution.
Liquid temperature =25.0°C, $\hat{d}_B \approx 3.7$ mm, $U_B \approx 0.20$ m/s.

2.4 Eddy diffusivities for fully developed turbulent bubble flow

In this paragraph sufficiently high liquid Reynolds number flows are considered such that fully developed turbulent motion occurs at a distance from the wall while a viscous sublayer exists in the immediate neughborhood of the wall. Then, the expressions for both the eddy diffusivities ϵ' and ϵ'' are examined so as to be able to describe the whole flow field from the vicinity of a smooth wall to the pipe center.

(i) ϵ'. In the Prandtl's theory to give the well-known logarithmic velocity distribution (Schlichting 1979), the eddy diffusivity is expressed as

$$\epsilon = \kappa^2 y^2 \frac{\mathrm{d}u}{\mathrm{d}y} = \nu\kappa y^+ \qquad [17]$$

where κ is the mixing length constant (= 0.4) and y^+ the dimensionless distance from the wall, $y^+ = yu^*/\nu$. In order to improve [17] for predicting accurately a flow in the region close to the smooth wall, van Driest (1956) suggests the following expression;

$$\begin{aligned}\epsilon &= \left\{1 - \exp\left(-\frac{y}{A}\right)\right\}^2 \kappa^2 y^2 \frac{\mathrm{d}u}{\mathrm{d}y} \\ &= \frac{\nu}{2}[-1 + \surd(1 + 4\kappa^2 y^{+2}\{1 - \exp(-y^+/A^+)\}^2)]\end{aligned} \qquad [18]$$

which has a form multiplying the Prandtl's formula [17] by the damping factor $\{1 - \exp(-y/A)\}^2$. It is known, however, that [18] is less accurate than the Reichardt's formula [9] in the determination of the velocity in the core region. When [9] is rewritten in terms of y^+, it becomes

$$\epsilon = \left\{1 - \frac{11}{6}\left(\frac{y^+}{R^+}\right) + \frac{4}{3}\left(\frac{y^+}{R^+}\right)^2 - \frac{1}{3}\left(\frac{y^+}{R^+}\right)^3\right\}\nu\kappa y^+ \qquad [19]$$

in which R^+ denotes $R^+ = Ru^*/\nu$. It is known that this formula is valid for the core flow region, but unfit for the wall region; as can be seen in [19], it tends to close with the Prandtl's expression [17] when the wall is approached.

Since [18] and [19] are excellent in the vicinity of the wall and in the core respectively, the combination of both formulas is expected to be suitable for the flow prediction of pipe flows over the entire cross section. Thus, the following expression is used for the eddy diffusivity ϵ' of the present study

$$\epsilon' = \left\{1 - \exp\left(-\frac{y^+}{A^+}\right)\right\}^2 \left\{1 - \frac{11}{6}\left(\frac{y^+}{R^+}\right) + \frac{4}{3}\left(\frac{y^+}{R^+}\right)^2 - \frac{1}{3}\left(\frac{y^+}{R^+}\right)^3\right\}\nu_L \kappa y^+ . \qquad [20]$$

Figure 3 compares the predicted velocity profiles by use of [20], taking $\kappa = 0.4$ and $A^+ = 16$, with the Laufer's experimental data for the flows of $\mathrm{Re} = 5 \times 10^4$ and 5×10^5 in a smooth pipe (Laufer 1954). In each case the calculated results are in close agreement with the measurements, and [20] is therefore considered to be satisfactory for the flow prediction from the wall surface to the pipe center.

(ii) ϵ''. As mentioned in section 2.3, the expression [10] for the eddy diffusivity ϵ'' indicating bubble agitation has been confirmed to be valid for flow prediction in the core region. However, it has been found that [10] gives high value of ϵ'' near the wall and, in other words, it evaluates the effect of bubble excessively. For further improvement on [10] it may be reasonable to take account of the damping effect of the smooth wall surface on the turbulent motion arising from bubble agitation. Accordingly, if the same damping factor applied to ϵ' is considered to be also

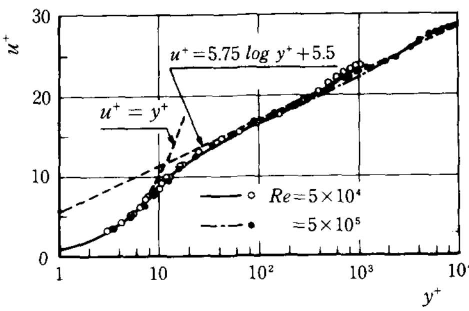


Figure 3. Comparison of the predicted velocity profile using [20] as to the eddy diffusivity with the experimental data of Laufer (1954). ——, Single-phase flow.

applicable to ϵ'', [10] can be modified as follows:

$$\epsilon'' = \left\{1 - \exp\left(-\frac{y^+}{A^+}\right)\right\}^2 k_1 \alpha \left(\frac{d_B}{2}\right) U_B \qquad [21]$$

in which A^+ is taken to be the same value as in [20], $A^+ = 16$. And, the empirical constant k_1 has been recommended as $k_1 = 1.2$ when the terminal velocity in the still liquid is applied to the bubble relative velocity U_B.

The deformation of bubble should be taken into consideration near the wall with regard to d_B in [21] since the scale of eddies generated by bubbles must depend on the dimension of bubble. From the observation of air bubbles in water flowing in a vertical channel (Sato *et al.* 1976), there are two kind of bubbles; sliding bubble just like to slide on the wall and coring bubble to flow away from the wall. The shape of a sliding bubble is different from an ellipsoid of a coring bubble. Figure 4 illustrates an example of the averaged shapes of sliding bubble at bulk water velocity of 1 m/s. A sliding bubble resembles a pear, i.e. a spherical cap at one side facing the core region and a cone with round top at the other side converging toward the wall. There is a narrow gap between the wall surface and the end of bubble, about $10 \sim 30\ \mu$m regardless of water velocity and bubble size. Taking into account this gap and the reduction of bubble dimension toward the wall, the following empirical expression for the bubble size is recommended as an approximation of d_B in [21]:

$$d_B = \begin{cases} 0 & \cdots\cdots\cdots 0 < y < 20\ \mu\text{m} \\ 4y(\hat{d}_B - y)/\hat{d}_B & \cdots 20\ \mu\text{m} < y < \hat{d}_B/2 \\ \hat{d}_B & \cdots\cdots\cdots \hat{d}_B/2 < y < R. \end{cases} \qquad [22]$$

in which $\hat{d}_B$ is the cross-sectional mean diameter of the bubbles.

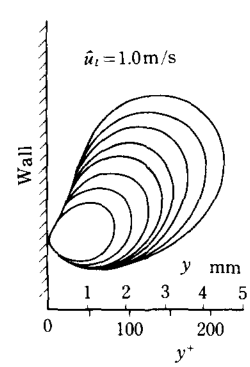


Figure 4. Averaged shapes of sliding bubbles at a bulk water velocity of 1.0 m/s (Sato *et al.* 1976).

3. HEAT TRANSFER

By analogy with a single-phase flow, it may be acceptable that the above-mentioned description for momentum transfer is extended to the heat transfer problem. If temperature fluctuation of the liquid phase is assumed to be split into the two independent components in a manner similar to the velocity fluctuations, the velocities and temperature can be expressed as

$$\left.\begin{aligned} u &= \bar{u} + u' + u'' \\ v &= \bar{v} + v' + v'' \\ T &= \bar{T} + T' + T'' \end{aligned}\right\} . \qquad [23]$$

Introducing [23] into the equation of energy for a two-dimensional system and then forming averages with respect to time, it is noticed that the two additional heat fluxes remain, $\rho c_p \overline{v'T'}$ and $\rho c_p \overline{v''T''}$. Furthermore, when the eddy diffusivities for heat, ϵ_H' and ϵ_H'', are defined by

$$\rho c_p \overline{v'T'} = -\rho c_p \epsilon_H' \frac{d\bar{T}}{dy} \qquad [24a]$$

$$\rho c_p \overline{v''T''} = -\rho c_p \epsilon_H'' \frac{d\bar{T}}{dy}, \qquad [24b]$$

the total heat flux involving also the heat conduction term can be written by

$$q = -\rho c_p (a + \epsilon_H' + \epsilon_H'') \frac{d\bar{T}}{dy} \qquad [25]$$

in which a is the thermal diffusivity.

Applying this to the present bubble flow problem, the following equation can be obtained, provided that gas bubbles can be treated as true voids;

$$\frac{q}{\rho_L c_L} = -(1-\alpha)(a_L + \epsilon' + \epsilon'') \frac{dT_L}{dy} \qquad [26]$$

where the subscript L denotes the liquid phase, and both eddy diffusivities ϵ_H' and ϵ_H'' for heat transfer are regarded as to be equal to those for momentum transfer ϵ' and ϵ'', respectively. When the wall heat flux q_w and the temperature difference between the wall and the bulk $(T_w - T_{Lb})$ are selected for reference, the following parameters can be defined;

$q^* = q/q_w$:	dimensionless heat flux	[27]
$T_L^* = (T_L - T_{Lb})/(T_w - T_{Lb})$:	dimensionless temperature	[28]
$h_{TP} = q_w/(T_w - T_{Lb})$:	heat transfer coefficient	[29]
$Nu = h_{TP}D/\lambda_L$:	Nusselt number.	[30]

Making use of these parameters, rearrangement of [26] gives the liquid temperature gradient in a dimensionless form;

$$\frac{dT_L^*}{dy^*} = -\frac{1}{2} \mathrm{Nu} \frac{q^*}{(1-\alpha)\left\{1 + \mathrm{Pr}_L\left(\frac{\epsilon' + \epsilon''}{\nu_L}\right)\right\}} \qquad [31]$$

where Pr_L is the Prandtl number of the liquid.

For both hydrodynamically and thermally developed flow, the dimensionless heat flux q^* can be written as

$$q^* = \frac{q}{q_w} = \frac{R}{r} \frac{\int_0^r (1-\alpha)u_L r \, \mathrm{d}r}{\int_0^R (1-\alpha)u_L r \, \mathrm{d}r} \quad [32]$$

for a vertical pipe, and

$$q^* = \frac{\int_0^r (1-\alpha)u_L \, \mathrm{d}r}{\int_0^R (1-\alpha)u_L \, \mathrm{d}r} \quad [33]$$

for parallel flat walls. Both [32] and [33] can be approximated by

$$q^* = 1 - y^* . \quad [34]$$

Equation [31] is the basic equation in determining the liquid temperature distribution, together with [20], [21] and [32] (or [34]) for ϵ', ϵ'' and q^*, respectively.

In addition, it seems likely that the description of mass transfer in a bubble flow can be given simply by replacing the temperature and the heat flux in the above equations with the concentration of mass and the mass flux, respectively.

4. NUMERICAL CALCULATIONS

4.1 *Liquid velocity distribution*

For prescribed pipe diameter D and liquid mass flow rate G_L, if the void fraction profile is specified, the liquid velocity distribution u_L and the frictional pressure gradient $\Delta P_f/\Delta x$ can be calculated numerically from [8], approximated by a finite difference expression. The iteration method is used to construct the velocity field such that the condition of continuity is satisfied, as shown in figure 5. The step by step procedure is as follows:

(1) Assign D, G_L and α-profile.

(2) Assume a value of τ_w.

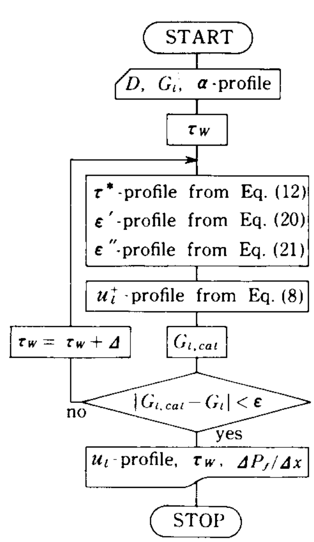


Figure 5. Explanatory diagram for the calculations of liquid velocity distribution and frictional pressure drop.

(3) Determine the corresponding distributions of τ^*, ϵ' and ϵ'' by use of [12], [20] and [21], respectively.

(4) Calculate the liquid velocity distribution u_L^+ from [8], using a backward finite-difference method under the boundary condition of $u_L^+ = 0$ at $y^* = 0$.

(5) Obtain the liquid mass flow rate $G_{L,\mathrm{cal}}$.

(6) Compare $G_{L,\mathrm{cal}}$ with the prescribed value of G_L.

(7) Repeat the foregoing process (steps 2–6) with the newly revised wall shear stress until a consistent velocity field is obtained. As a result, the corresponding value of frictional pressure gradient (or wall shear stress) can also be determined.

The nodes have to be closely arranged particularly near the wall for a few of them so as to be in the viscous sublayer, but may be arranged at a considerable distance apart in the core region, e.g. $\Delta y^* \approx 0.05$. It was found that the satisfactory velocity field usually can be determined within 5–8 iterations.

4.2 *Liquid temperature distribution*

If the wall heat flux q_w and the bulk liquid temperature T_{Lb} are given in addition to the velocity field determined in the above procedure, the heat transfer problem can be solved numerically. In calculating the liquid temperature distribution, the basic differential equation [31] is also approximated by a finite difference expression. And, for example, the iteration method shown in figure 6 can be recommended in such a way so as to find out a compatible wall temperature. The process is continued until the calculated bulk liquid temperature approaches satisfactorily to the prescribed value. As a result, a consistent temperature field and a heat transfer coefficient are obtained.

5. CONCLUSIONS

A theory for momentum transfer of a two-phase bubble flow in channels has been developed, based on the assumption that the two kinds of turbulence are present in the liquid phase; inherent wall turbulence independent of bubble relative motion and additional turbulence caused by bubble agitation. It is then possible to predict theoretically the liquid velocity distribution and frictional pressure gradient for a given flow when its void fraction profile is known.

The theory has been extended to heat transfer problem; the additional turbulent heat flux has been regarded as to be subdivided further into the two components independent of and

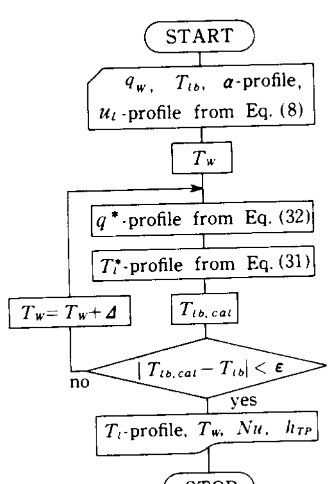


Figure 6. Explanatory diagram for the calculations of liquid temperature distribution and heat transfer coefficient.

dependent on bubble agitation. The result of the analysis leads to the theoretical calculation of liquid temperature distribution and heat transfer coefficient, provided that eddy diffusivity for heat transfer is equal to that for the momentum transfer.

A method for the numerical calculations of the velocity and temperature fields has been presented for given pipe diameter, liquid mass flow rate, heat flux and bulk temperature.

For a flow in the absence of the wall turbulence, $Re_L < 2300$, the agreement between the calculated and measured values is quite reasonable. However, to gain confidence in the proposed theory there is still a need to make comparisons with available experiments in particular for fully developed turbulent bubble flows, both wall turbulence and bubble agitation being significant.

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